DESIGN OF LAMINAR AND FIBROUS COMPOSITES WITH GIVEN CHARACTERISTICS

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The problem of designing composites with given sets of average characteristics [1, 2] which is of considerable practical interest, represents an inverse problem in the general case that is referred to synthesis problems according to a common classification [3]. It is also similar to optimal control problems for partial differential equations [4] and can be reduced to it for searching for particular solutions (see [5]). Investigation of the design problem in the general case is constrained by the small number of theoretical results [5].

Meanwhile, the presence of explicit expressions for average characteristics for the classes of composites used extensively in practice (laminar and fibrous) and the results in [2, 6, 7] (in the part of local stress estimates) permits reducing the design problem to particular cases of integral equations of the first kind for which methods of solution will successfully be developed that are sufficiently effective for utilization in the solution of practical problems.

1. DESIGN OF LAMINAR COMPOSITES WITH GIVEN AVERAGE CHARACTERISTICS

Let a composite be formed by periodically alternating thin (thickness $\varepsilon \ll 1$) layers of homogeneous isotropic materials parallel to the Ox_1x_2 plane. Then the average characteristics [1, 2, 8] of composites of laminar configuration (specific gravity $\hat{\rho}$, pliability tensor $\hat{H}_{ijk\ell}$, thermal expansion coefficients $\hat{\beta}_{ij}$, etc.) are expressed in terms of the local characteristics [specific gravity $\rho(x_3/\varepsilon)$, Young's modulus $E(x_3/\varepsilon)$, Poisson ratio $\nu(x_3/\varepsilon)$, the coefficient of thermal expansion $\beta(x_3/\varepsilon)$] by formulas [1, 2, 9]

$$\begin{gathered}\widehat{\rho}=\langle\rho\rangle,\ \widehat{H}_{3333}=\left\langle\frac{(1+\nu)(1-2\nu)}{E(1-\nu)}\right\rangle+\frac{2\langle\nu/(1-\nu)\rangle^2}{\langle E/(1-\nu)\rangle}\quad\text{etc.},\\ \widehat{\beta}_{33}=\widehat{H}_{3333}\frac{\left\langle\frac{\beta(1+\nu)}{1-\nu}\right\rangle}{\left\langle\frac{(1+\nu)(1-2\nu)}{E(1-\nu)}\right\rangle}+2H_{3322}\left[\left\langle\frac{E\beta}{(1+\nu)(1-2\nu)}\right\rangle-\right.\\ \left.-\left\langle\frac{2\nu^2E\beta}{(1-\nu^2)(1-2\nu)}\right\rangle+\frac{\left\langle\frac{\nu}{1-\nu}\right\rangle\left\langle\frac{(1+\nu)\beta}{1-\nu}\right\rangle}{\left\langle\frac{(1+\nu)(1-2\nu)}{E(1-\nu)}\right\rangle}\right]\text{etc.},\end{gathered}\tag{1.1}$$

where $\langle\cdot\rangle=\int_0^1\cdot\,dy\ (y=x_3/\varepsilon)$ is the average over a period of the composite structure.

We do not present all the formulas to evaluate the quantities designated, they are well known, a sufficiently complete listing can be found in [10]. For our purposes it is just essential that (1.1) have the form

$$(\widehat{\rho},\widehat{H}_{ijkl},\widehat{\beta}_{ij})=F\left(\langle\rho\rangle,\left\langle\frac{(1+\nu)(1-2\nu)}{E(1-\nu)}\right\rangle,\ldots\right)\tag{1.2}$$

(F is an algebraic function, and ... denotes independent integral functionals [besides those mentioned explicitly) in (1.1)].

PROBLEM OF DESIGNING A COMPOSITE OF ONE-DIMENSIONAL CONFIGURATION WITH A GIVEN SET OF AVERAGE CHARACTERISTICS

This is formulated as follows: 1. Is (1.2) solvable in the given class of functions? 2. If it is solvable, indicate the set of its solutions.

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Extraction of the solvability question is typical for incorrect problems to which the one we examine belongs.

Remark 1. The selection of the class of functions $\mathcal{U}$ is by starting from the existing composite production technology. For instance, the class of functions

$$\mathcal{U}_\infty = \left\{\begin{array}{l}\bar{u}(y) \in L_\infty([0,\ 1]):\ \text{all the} \\ \text{integrands in (1.1) are defined}\end{array}\right\}$$

corresponds to application of materials with any combinations of mechanical characteristics and any kinds (continuous, piecewise-continuous, etc.) of their distributions on the composite structure as components. The class of functions $\mathcal{U}_\infty$ is used for theoretical investigations of the problem. We do not have such a class of materials available in practice. The class of functions

$$\mathcal{U}_d = \left\{\begin{array}{c}\bar{u}(y) \in \mathcal{U}_\infty:\ \bar{u}(y) \in \{\rho_\alpha, E_\alpha, \nu_\alpha, \beta_\alpha\}_{\alpha=1}^m \\ \text{for almost all}\quad y \in [0,\ 1]\end{array}\right\}$$

corresponds to utilization of a finite number of materials m for creating a composite. This is the case most widespread in practice.

Remark 2. All solutions of the problem (or at least as large a number of them as possible) are required in formulating the synthesis problem. This condition is combined with engineering requirements since it is always desirable to have the greatest number of different designs of a composite with the necessary properties in order to select the most technological ones.

It is convenient to start from the following problem rather than directly from (1.2) for its solution. Let us replace the functionals $\langle\rho\rangle$, ... in (1.2) by the variables $y_1, \ldots, y_n$. We consequently obtain an algebraic system (the system S [10]) in $y_1, \ldots, y_n$ ($\hat{\rho}$, $\hat{H}_{ijk\ell}$, $\hat{\beta}_{ij}$ are given). Let Y denote the set of solutions of the system S (its solution is not unique in the general case). Afterwards (1.2) reduces to

$$\int_0^1 \bar{f}(\bar{u}(y))\, dy = \bar{y},\quad \bar{y} \in Y \subset R^n, \tag{1.3}$$

where $\bar{u}(y) = (\rho(y), E(y), \nu(y), \beta(y))$ is the set of local characteristics in the period of the composite structure, the desired function. Integrands (under the symbol $\langle\ \rangle$) in (1.1) are denoted by $\bar{f}(\bar{u})$. For laminar composites the functions $\bar{f}(\bar{u})$ are presented completely in [8].

Equation (1.3) in the class of functions $\mathcal{U}_d$ of laminar composites goes over into

$$\sum_{\alpha=1}^m \bar{y}_\alpha \lambda_\alpha = \bar{y},\quad \lambda_\alpha \geqslant 0,\quad \sum_{\alpha=1}^m \lambda_\alpha = 1,\quad \bar{y} \in Y \tag{1.4}$$

for $\{\lambda_\alpha\}$ the volume content of components in the composite (λ_α is the volume content of the α-th material in the composite). The notation $\bar{y}_\alpha = \bar{f}(\bar{u}_\alpha)$ is used, where $\bar{u}_\alpha = (\rho_\alpha, E_\alpha, \nu_\alpha, \beta_\alpha)$ is the set of mechanical characteristics of the α-th material.

DOMAIN OF POSSIBLE VALUES OF THE AVERAGE CHARACTERISTICS

Both theoretically and practically, the question of what average characteristics can be ascribed, in principle, to laminar composites, is of interest. In particular, the assertion that properties different from the properties of the components can be adduced became commonplace in application to composites. How can this be manifest in application to laminar composites?

From the mathematical point of view, the solution of the formulated problem reduces to calculating the image of the set $\mathcal{U}$ upon mapping (1.1). For the case $\mathcal{U} = \{\bar{u}(y) \in \mathcal{U}_\infty$: $\rho(y), E(y), \nu(y), \beta(y) > 0$ for almost all $y \in [0, 1]\}$ (under the simplifying assumption about agreement between the Poisson ratios of the components) this problem is solved in [11] by optimal control methods.

There results from the solution mentioned: a) Composites of the type under consideration can have the following average characteristics: $\hat{\rho} = X$ $(X > 0)$; the average Young's moduli $\hat{E}_i$, the Poisson ratios $\hat{\nu}_{ij}$, and the shear moduli $\hat{G}_{ij}$:

$$\widehat{E}_1 = \widehat{E}_2 = x, \quad \widehat{E}_3 = \frac{(1-\nu)\,x}{(1+\nu)(1-2\nu)\,xy + 2\nu^2},$$
$$\widehat{\nu}_{12} = \nu, \quad \widehat{\nu}_{13} = \widehat{\nu}_{23} = \frac{\nu(1-\nu)}{(1+\nu)(1-2\nu)\,xy + 2\nu^2},$$
$$\widehat{G}_{12} = \frac{2x}{1+\nu}, \quad \widehat{G}_{13} = \widehat{G}_{23} = \frac{2}{(1+\nu)\,y};$$
$$\widehat{\beta}_{11} = \widehat{\beta}_{22} = \frac{t}{x}, \quad \widehat{\beta}_{33} = \frac{1+\nu}{1-\nu}z - \frac{2\nu}{1-\nu}\frac{t}{x}$$

$(x > 0,\ y > 1/x,\ z > 0,\ t > 0)$. The variables X, x, y, z, t in the domains mentioned for them take on independent values. b) Any average characteristics of composites with continuous, piecewise-continuous, etc., distributions of the local characteristics can be obtained as average characteristics of composites of laminar construction.

Returning to the formula presented for $\hat{\beta}_{33}$, it is easy to note that the domain of possible values of $\hat{\beta}_{33}$ is $(-\infty, +\infty)$. This means that the coefficient of thermal expansion of the composite (formed from components with positive (!) coefficients of thermal expansion, see above) can be negative. Composites based on actually existing materials are mentioned in [11], that possess this property. The remaining average characteristics do not allow examples of such impressive qualitative distinctions in the properties of the composite and its components (although quantitative distinctions can be quite significant).

This case is interesting also in connection with the utilization of different, often simplified, models of composites. It should be noted that models of the level of the mixture rule are not able to perceive the effect presented above.

As regards the assertion of Sec. b, it is useful in practice since it yields an answer to the question of whether continuous, etc., distributions of local characteristics can adduce any new properties to a composite as compared with the properties of traditional (highly technological in fabrication) laminar composites. As we see, all possible properties can be realized in the class of laminar composites. An analogous result also holds when taking account of the strength properties of materials.

MATHEMATICAL METHODS OF SOLVING PROBLEMS (1.3), (1.4)

It was noted above that utilization of the class of functions $\mathcal{U}_\infty$ is not adequate for situations that actually occur when only a certain limited set of materials can be used to create a composite. The class of functions

$$U = \{\bar{u}(y) \in \mathcal{U}_\infty\colon \bar{u}(y) \in V \text{ for almost all } y \in [0, 1]\},$$

corresponds to this case that is most extensively encountered in practice, where V is compact in R^n (n depends on the number of component characteristics in the formula for the average characteristics). When the set V is formed by a finite set of points $(V = \{\rho_\alpha, E_\alpha, \nu_\alpha, \beta_\alpha\}_{\alpha=1}^m)$, we have $U = \mathcal{U}_d$.

Proposition 1. a) For $m \geq n + 1$ the image of the set U during the mapping of (1.3) is a convex hull of the set

$$\Sigma = \{\bar{x} \in R^n\colon \bar{x} = \bar{f}(\bar{u}),\ \bar{u} \in V\}.$$

b) Any point belonging to the image U during the mapping of (1.3) can be obtained as the value of (1.3) on a piecewise-constant function taking on not more than n + 1 different values (i.e., again all possible average characteristics are realized in the class of laminar composites).

Remark 3. When using a finite number of components [see (1.4)], the set Σ is a finite set of points $\Sigma = \{\bar{y}_\alpha\}_{\alpha=1}^m$ [$\bar{y}_\alpha$ are defined in the clarification of (1.4)] while $\operatorname{conv}\Sigma = \operatorname{conv}\{\bar{y}_\alpha\}_{\alpha=1}^m$ is a convex polyhedron. In this case the problem occurs of convex combinations [see (1.4)] [10, 12]: 1. Does this point $\bar{y}$ belong to the polyhedron $\operatorname{conv}\Sigma = \operatorname{conv}\{\bar{y}_\alpha\}_{\alpha=1}^m$. 2. If it belongs, indicate all coefficients of convex combinations of points $\{\bar{y}_\alpha\}_{\alpha=1}^m$ yielding the point $\bar{y}$.

Remark 4. We arrive at the same problem upon discretization of the problem (1.3) during its numerical solution, see [8].

Let us consider the nondegenerate simplexes $\{P_1, \ldots, P_M\}$ of the polyhedron $\{\bar{y}_\alpha\}_{\alpha=1}^m$ containing the point $\bar{y}$. A single solution of the problem (1.4) [11] corresponds to each simplex P_η and the point $\bar{y}$, and let us denote it by $\bar{s}_\eta(\bar{y})$. Let us note that the number M of such simple solutions is finite. Let $\Lambda(\bar{y})$ denote the set of all solutions of the problem (1.4).

Proposition 2.

$$\Lambda(\bar{y}) = \operatorname{conv}\{\bar{s}_\eta(\bar{y})\}_{\eta=1}^M.$$

Let us define the "residual" between the two polyhedra A, B $\subset R^n$ as the number

$$(A, B) = \max_{\bar{x}\in B\setminus A}\min_{\bar{y}\in A}|\bar{x}-\bar{y}| + \max_{\bar{x}\in A\setminus B}\min_{\bar{y}\in B}|\bar{x}-\bar{y}|. \tag{1.5}$$

Geometrically this quantity is of the order of the "residual thickness" of the polyhedra A and B equal to $(A\setminus B) \cup (B\setminus A)$ [hence two components in (1.5)]. Giving the function (1.5) in the set of polyhedra in R^n transforms it into a topological space.

Proposition 3. Let $\bar{y}_i \to \bar{y}$ as $i \to \infty$, where $\{\bar{y}_i, i \to \infty\}$, $\bar{y} \in \operatorname{conv}\{\bar{y}_\alpha\}_{\alpha=1}^m$. Then $(\Lambda(\bar{y}_i), \Lambda(\bar{y})) \to 0$ as $i \to \infty$.

The proofs of Propositions 1 and 2 are presented in [12]. The proof of Proposition 3, assuring the possibility of discretization of the set Y for practical calculations, is of purely mathematical nature, in which connection it is not presented here.

COROLLARY. The set of solutions of the problem of designing laminar composites with a given set of average characteristics is this: the α-th material enters the composite in the volume content

$$\lambda_\alpha = \sum_{\eta=1}^{M} \mu_\eta s_{\eta\alpha}(\bar{y}), \tag{1.6}$$

where $\{\mu_\eta\}$ are arbitrary numbers satisfying the condition $\mu_\eta \geq 0$; $\sum_{\eta=1}^{M}\mu_\eta = 1$ ($s_{\eta\alpha}(\bar{y})$, $\alpha = 1, \ldots, n$ are coordinates of the simplicial solution $\bar{s}_\eta(\bar{y})$).

TAKING ACCOUNT OF THE STRENGTH OF COMPONENTS FOR THE DESIGN

To include the strength characteristics in the considerations, it is necessary to have an average criterion of the strength of a composite, a strength criterion in terms of the average stress or strains (stress and strain determinable from the solution of the problem of body deformation with average characteristics upon application of the same load as to the initial body are understood to be such). Such criteria can be obtained on the basis of using the C^1-asymptotic of local stresses or strains expressed in terms of average quantities. There are no such asymptotics in the general case. For ordinary differential equations the C^1-asymptotic of the averaging method is obtained in [7, 15] (in application to the problem under consideration): $\sigma_{ij}^\varepsilon \sim \Phi_{ijk\ell}(\hat{H}_{ijk\ell}, \hat{\beta}_{ij}, E(y))\sigma_{k\ell}$. The specific form of the function $\{\Phi_{ijk\ell}\}$ is presented in [10, 16]. Let the strength criteria of the component materials be $0 \leq f(\sigma_{ij}^\varepsilon, E) \leq \sigma(E)$, where σ_{ij}^ε are the local stresses, E is used as a material indicator (when using a finite number of materials $f_\alpha(\sigma_{ij}^\varepsilon) \leq \sigma_\alpha$ is the strength criterion of the α-th material). Substituting the asymptotic σ_{ij}^ε in the strength criterion (under the condition of boundedness of $\partial f/\partial\sigma_{ij}^\varepsilon$) results in an average strength criterion (see [16, 17] for greater detail)

$$M(\sigma_{ij}, E) \equiv \max_{y\in[0,1]} F(\hat{H}_{ijkl}, \hat{\beta}_{ij}, E(y), \sigma_{ij}) \leqslant 1,$$

and in the case of materials of laminar configuration

$$M(\sigma_{ij}, E) \equiv \max_{\alpha \in \Sigma} F_\alpha(\widehat{H}_{ijkl}, \widehat{\beta}_{ij}, \sigma_{ij}) \leqslant 1 \qquad (1.7)$$

(Σ is the set of numbers of materials actually in the composite). The functions F, $\{F_\alpha\}$ are determined by the requisite average characteristics and the component characteristics (see examples in [10, 16]).

DESIGN OF LAMINAR COMPOSITES WITH GIVEN DEFORMATION-STRENGTH CHARACTERISTICS

Solve the problem (1.4) under the additional conditions

$M(\sigma_{ij}, E) \leq 1$ (the composite endures a given average load)

or

$M(\sigma_{ij}, E) \to \min$ (the composite has the highest safety factor, the strongest composite). The methods of solving the problem formulated are exposed in detail in [16, 17].

MAXIMALLY STRONG AND EQUALLY STRONG DESIGNS

Let us note that the results of [16] permit making the following deduction. Let $(E^*) = \{\lambda_\alpha^*\}$ be the design of the strongest composite in the above-mentioned sense, and $(E) = \{\lambda_\alpha\}$ the design of a certain equally strong composite (a composite for which the strength criteria of all the components are spoiled simultaneously). Then a strict inequality is realized, as a rule in the evident nonstrict inequality $M(\sigma_{ij}, E^*) \leq M(\sigma_{ij}, E)$ for any equally strong design for laminar composites. Thus, equally strong designs do not realize the greatest strength of a composite although the criterion (1.7) is the only rupture criterion "at the first crack" [spoilage of (1.7) generally implies rupture of just certain layers].

LAMINAR COMPOSITES OF MAXIMAL SPECIFIC STRENGTH

The problem considered below, which is of practical interest, illustrates the "theoretical" means for the occurrence of composites as piecewise-constant solution of the design problem in the absence of solutions in the form of a constant. An analogous fact was remarked in application to another problem in [18].

Let an average stress $t\sigma_{ij}{}^0$, $\sigma_{ij}{}^0 = 0$ for i or j = 3 be applied to a material (proportional loading in the plane of the layers). The asymptotic of the local stresses in this case is $\sigma_{ij}^\varepsilon = \frac{E_\alpha}{\langle E \rangle} t\sigma_{ij}^0$ in the layer occupied by the α-th material. Let the functions $f_\alpha \geq 0$ be homogeneous functions of the first degree. The average strength criterion

$$\max_{\alpha \in \Sigma} f_\alpha \left(\frac{E_\alpha}{\langle E \rangle} t\sigma_{ij}^0 \right) \leqslant 1$$

yields the following value of the parameter t for which rupture starts

$$t^* = \langle E \rangle \Big/ \max_{\alpha \in \Sigma} f_\alpha (E_\alpha \sigma_{ij}^0)$$

because of the homogeneity of the functions. The specific strength of the composite is $t^*/\hat{\rho}$. Then the form

$$\frac{\widehat{\rho}}{t^*} = \frac{\langle \rho \rangle}{\langle E \rangle} \max_{\alpha \in \Sigma} f_\alpha (E_\alpha \sigma_{ij}^0) \to \min. \qquad (1.8)$$

can be adduced to the problem of maximizing the specific strength. Let us order the material data so that their corresponding numbers $m_\alpha = f_\alpha(E\sigma_{ij}{}^0)$ would be arranged in increasing order. Then $\max_{\alpha = 1, \dots, K} m_\alpha = m_K$ outside the dependence of the real entrance of the α-th ($\alpha < K$) material in the composite. Consequently, the problem (1.8) is written as

$$\frac{\langle\rho\rangle}{\langle E\rangle} m_K \to \min = \min_K.$$

for the set of materials with numbers $\alpha \in \{1, \ldots, K\}$. The quantities $(\langle E\rangle, \langle\rho\rangle)$ (see Remark 3) can take on values filling the polyhedron $\mathrm{conv}\{(E_\alpha, \rho_\alpha);\ \alpha = 1, \ldots, K\}$ independently. Taking this into account the problem reduces to the following (for a given K):

$$y/x \to \min = \min_K; \tag{1.9}$$

$$(x, y) \in R_K = \mathrm{conv}\{(E_\alpha, m_K\rho_\alpha);\ \alpha = 1, \ldots, K\}. \tag{1.10}$$

The problem (1.9), (1.10) has a solution and $\min_K$ equals the minimal angular coefficient of lines $y = kx$ having a common point with the polyhedron R_K.

Remark 5. Because of the convexity of R_K the line mentioned certainly passes through its apex. Therefore, the problem (1.9), (1.10) possesses a solution of the form $(x, y) = (E_\beta, m_K\rho_\beta)$ or $(\langle E\rangle, \langle\rho\rangle) = (E_\beta, \rho_\beta)$ which corresponds to a homogeneous material.

Furthermore, selecting $K \in \{1, \ldots, m\}$ (we denote it by K_*) for which k has the minimal value, we solve the problem

$$\sum_{\alpha=1}^{K_*} E_\alpha\lambda_\alpha = x, \quad \sum_{\alpha=1}^{K_*} \rho_\alpha\lambda_\alpha = y/\min_{K_*}, \quad \lambda_\alpha \geqslant 0, \quad \sum_{\alpha=1}^{K_*} \lambda_\alpha = 1$$

to determine the volume contents of the components.

COROLLARY. A material possessing the maximal specific strength among those fabricated on the base of m given homogeneous materials is one of these homogeneous materials. The corollary results from the Remark 4.

Let the same problem of designing a maximal strength composite be supplemented by a constraint on the specific gravity $\langle\rho\rangle \le \rho_0$. In this case we arrive at a problem of minimizing (1.9) under the condition

$$(x, y) \in S_K = R_K \cap \{y \leqslant m_K\rho_0\}.$$

The apices of the polyhedra S_K may already not agree with points of the form $(E_\alpha, m_K\rho_\alpha)$

COROLLARY. The problem under consideration with the constraint on the specific gravity cannot have a solution for a homogeneous material but always has a solution corresponding to a laminar composite. Utilization of many constraints occurring in practice results in analogous results.

2. DESIGN OF COMPOSITES BASED ON HIGH MODULE FIBERS

Let us examine composite materials formed by stacking layers of parallel fibers (the so-called "prepreg" technology [19]). Let us consider the layers parallel to the Ox_1x_2 plane (this does not diminish the generality), let φ_α denote the angle between the fiber axes of the α-th layer and the Ox_1 axis. Then [2] the average stiffness characteristics $\{\hat{a}_{ijk\ell}\}$ in the Ox_1x_2 plane ($i, j, k, \ell = 1, 2$) and the local stresses σ_{ij}^ε in the fibers are

$$\hat{a}_{ijkl} = Es \sum_{\alpha=1}^{M} \gamma_i^\alpha\gamma_j^\alpha\gamma_k^\alpha\gamma_l^\alpha\lambda_\alpha; \tag{2.1}$$

$$\sigma_{ij}^\varepsilon = E\gamma_i^\alpha\gamma_j^\alpha \sum_{k,l=1}^{2} \gamma_k^\alpha\gamma_l^\alpha e_{kl} \tag{2.2}$$

in the α-th fiber layer.

The average thermal expansion coefficients are

$$\hat{\beta}_{ij} = \beta \sum_{\alpha=1}^{M} \gamma_i^\alpha\gamma_j^\alpha\lambda_\alpha. \tag{2.3}$$

The term $\theta E\beta\gamma_i^\alpha\gamma_j^\alpha$ is appended to the right side of (2.2) when taking account of thermal expansion.

We use the notation: E, β are Young's modulus and the thermal expansion coefficient of the fibers, s is the volume fiber content in the composite, θ is the temperature, $\{\gamma_i^\alpha\}$ are the direction cosines of the fiber axes of the α-th layer, M is the number of bonding families per period of the composite structure, λ_α is the specific (referred to s) fiber content in the α-th bonding layer. The average stresses σ_{ij} and strains e_{ij} are defined as before.

The local stress asymptotics in the binder cannot be obtained in explicit form, in contrast to the preceding cases, since the problem of binder deformation by stiff fibers (of the type of the "stiff" problem [20]) must be solved for their determination, which is realizable only numerically in practice. Meanwhile, by using explicit approximate solutions, estimates can be obtained for the local stresses in the binder and sufficient conditions for the binder strength on their basis. The analysis made in [21] for the strain of a soft binder clarified two characteristic kinds of its deformation: interlayer and interfiber. Upon going over to an average strength criterion, this results in the occurrence of a number of criteria, each of which corresponds to its own kind of binder rupture at the microlevel:

Strength criterion in interfiber stresses

$$\max_{\alpha\in\Sigma} f_1(\varphi_\alpha, e_{ij}) \leqslant 1,$$

Strength criterion in interlayer stresses

$$\max_{\alpha\in\Sigma} f_2(\varphi_\alpha, \varphi_{\alpha+1}, e_{ij}) \leqslant 1.$$

The method of constructing the functions f_1 and f_2 is described in [17]. As regards the fiber strength criterion, it is obtained, as above, by substituting the asymptotic (2.2) into the strength condition for the fiber material

$$\max_{\alpha\in\Sigma} f_b(\varphi_\alpha, e_{ij}) \leqslant 1 \tag{2.4}$$

(Σ is the set of numbers of fiber layer stacking angles actually being utilized in the composite design).

The substitution $\{e_{ij}\} = \{\hat{a}_{ijk\ell}\}^{-1}\{\sigma_{ij}\}$ permits writing the strength criterion in terms of the average stresses ($\{\hat{a}_{ijk\ell}\}$ are given by (2.1), in the thermoelastic case (2.3) must be taken into account).

It should be kept in mind when writing the average strength criteria that $\gamma_1^\alpha = \cos\varphi_\alpha$, $\gamma_2^\alpha = \sin\varphi_\alpha$, $\gamma_3^\alpha = 0$. Consequently, all the sums in the right sides of (2.1) and (2.3) are expressed in terms of four independent functionals

$$R_1(\overline{\varphi}) = \sum_{\alpha=1}^{M} \lambda_\alpha \cos^4\varphi_\alpha, \quad R_2(\overline{\varphi}) = \sum_{\alpha=1}^{M} \lambda_\alpha \sin^4\varphi_\alpha; \tag{2.5}$$
$$R_3(\overline{\varphi}) = \sum_{\alpha=1}^{M} \lambda_\alpha \sin\varphi_\alpha \cos^3\varphi_\alpha, \quad R_4(\overline{\varphi}) = \sum_{\alpha=1}^{M} \lambda_\alpha \sin^3\varphi_\alpha \cos\varphi_\alpha,$$

where

$$\lambda_\alpha \geqslant 0, \ \sum_{\alpha=1}^{M} \lambda_\alpha = 1. \tag{2.6}$$

The system S obtained by replacing $R_1(\overline{\varphi}), \ldots, R_4(\overline{\varphi})$ in (2.1) by $y_1, \ldots, y_4$ is solved explicitly

$$y_1 = \frac{\widehat{a}_{1111}}{Es}, \quad y_2 = \frac{\widehat{a}_{2222}}{Es}, \quad y_3 = \frac{\widehat{a}_{1112}}{Es}, \quad y_4 = \frac{\widehat{a}_{1222}}{Es} \tag{2.7}$$

[the solvability condition $\widehat{a}_{1212} = \widehat{a}_{1122} = (1/2)(Es - \widehat{a}_{1111} - \widehat{a}_{2222})$ occurs here].

Remark 6. The average thermal expansion coefficients are expressed in terms of the functionals $J_1(\bar{\varphi})=\sum_{\alpha=1}^{M}\lambda_\alpha \cos^2\varphi_\alpha$, $J_2(\bar{\varphi})=\sum_{\alpha=1}^{M}\lambda_\alpha \sin^2\varphi_\alpha$ reducible to (2.5). Assignment of $\hat{\beta}_{ij}$ imposes additional conditions of the solvability of the system S.

Let us note that in the case under consideration the fiber stacking angles take on a finite number of values but neither their number M nor the angles themselves are fixed. In practice a constraint on the fiber stacking angles of the form

$$\varphi \in \Phi_\alpha = [a_1, b_1] \cup \ldots \cup [a_r, b_r] \tag{2.8}$$

occurs.

PROBLEM OF DESIGNING A FIBROUS COMPOSITE WITH GIVEN AVERAGE CHARACTERISTICS

1. Are the equations

$$R_i(\bar{\varphi}) = y_i, \; i = 1, 2, 3, 4, \tag{2.9}$$

solvable under the conditions (2.8) and (2.5)?

2. If they are solvable, then indicate the set of solutions.

MATHEMATICAL METHODS OF SOLVING DESIGN PROBLEMS

Application of the methods of [14] results in an analog to Proposition 1.

Proposition 4. a) For M ≥ 5 the image of the set $\mathscr{V} = \left\{(\varphi_\alpha, \lambda_\alpha)\colon \varphi_\alpha \in \Phi_\alpha, \lambda_\alpha \geqslant 0, \sum_{\alpha=1}^{M} \lambda_\alpha = 1\right\}$ is the convex hull of the set

$$\Gamma = \{\bar{x} \in R^4\colon x_1 = \cos^4\varphi, \; x_2 = \sin^4\varphi,$$
$$x_3 = \sin\varphi\cos^3\varphi, \; x_4 = \sin^3\varphi\cos\varphi, \; \varphi \in \Phi_\alpha\}.$$

b) Any point belonging to the image of the set $\mathscr{V}$ can be obtained as the value of the functions $R_1(\bar{\varphi}), \ldots, R_4(\bar{\varphi})$ for M = 5.

COROLLARY. To obtain a fibrous composite of the type under consideration with any possible sets of average stiffness characteristics $\{\hat{a}_{ijk\ell}\}$ (and the thermal expansion $\{\hat{\beta}_{ij}\}$) it is sufficient to use not more than five families of bonding fibers.

Remark 7. Symmetric fiber stackings $\{\pm\varphi_\alpha\}$ characterized by the relationship $\varphi_\alpha = -\varphi_{M+1-\alpha}$, $\lambda_\alpha = \lambda_{M+1-\alpha}$, $\alpha = 1, \ldots, M/2$ (M even) are often used in practice. In this case $R_3(\bar{\varphi}) = R_4(\bar{\varphi}) \equiv 0$ and the dimensionality of (2.9) is lowered to two [i = 1, 2 in (2.9)]. The line Γ takes the form

$$\Gamma = \{\bar{x} \in R^2 : x_1 = \cos^4\varphi, \; x_2 = \sin^4\varphi, \; \varphi \in \Phi_\alpha\}.$$

Graphical methods clearly illustrating the solution technique utilized can be used to solve the problem.

Let us examine the case when the possible stacking angles take on a finite number of values $\{\varphi^\nu\}_{\nu=1}^{m}$. The problem (2.9) and (2.6) reduces to a problem about convex combinations (1.4) with

$$\bar{y}_\alpha = (\cos^4\varphi^\alpha, \; \sin^4\varphi^\alpha, \; \sin\varphi^\alpha\cos^3\varphi^\alpha, \; \sin^3\varphi^\alpha\cos\varphi^\alpha)$$

(or $\bar{y}_\alpha = (\cos^4\varphi^\alpha, \sin^4\varphi^\alpha)$ in the case of symmetric stackings). Correspondingly, for m ≥ 5, Propositions 2 and 3 hold.

We considered the volume fiber content s fixed above. Let us now consider the following problem.

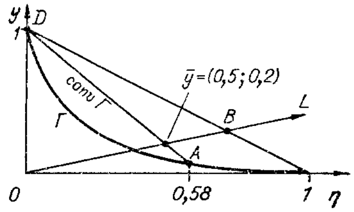

Fig. 1

DESIGN OF A COMPOSITE WITH GIVEN $\hat{a}_{ijk\ell}$, $\hat{\beta}_{ij}$ WHEN USING A MINIMAL VOLUME OF FIBERS

Solve the problem $s \to \min$ while satisfying conditions (2.9), (2.6), and (2.8).

The problem formulated is easily reduced to the preceding one. Let $\bar{y}_0$ be a certain solution of the system S (2.7) (s_0 is the volume fiber content corresponding to $\bar{y}_0$) for which the problem (2.9), (2.6), and (2.8) is solvable. The inclusion of s among the variables results in the problem (2.9), (2.6) and (2.8) with right side of (2.9) equal to $\bar{y}_0 s_0/s$. The points $L = \{\bar{y}_0 s_0/s : s \in [s_0, 0)\}$ form a ray. In conformity with Sec. a) of Proposition 4 it is sufficient to find the point of intersection of the ray L with the convex set Γ which will yield the desired value of s. This is a standard convex programming problem.

Example 1. Let it be required to create a composite with the average stiffnesses $\hat{a}_{1111} = 0.5 \cdot 10^{11}$ Pa, $\hat{a}_{2222} = 0.2 \cdot 10^{11}$ Pa from a fiber with the Young's modulus $E = 1.25 \cdot 10^{11}$ Pa for a volume fiber content of $s = 0.8$. There are no constraints on the stacking angles $\Phi_\alpha = [0, \pi]$. Symmetric stacking schemes of the type $\{\pm\varphi_\alpha\}$ are used. The arc

$$\Gamma = \{(\cos^4\varphi, \sin^4\varphi) : \varphi \in [0, \pi]\} = \{(\eta, (1 - \sqrt{\eta})^2) : \eta = \cos^4\varphi \in [0, 1]\}$$

is represented in Fig. 1. Displayed there is its convex hull conv Γ. The solution of the system S in the case under consideration is $y_1 = 0.5$, $y_2 = 0.2$. The point $\bar{y}$ belongs to conv Γ. Therefore, the design problem is solvable. There are infinitely many designs to create the composite. For instance, the point $\bar{y}$ can be obtained as a convex combination of the points A and D. The stacking angles of the fiber layers are $\pm\varphi_1 = \pm\arccos 0 = \pm 90^\circ$ and $\pm\varphi_2 \approx \pm\arccos\sqrt[4]{0.58} \approx \pm 31^\circ$ (since $\eta_1 = 0$, $\eta_2 \approx 0.58$, see Fig. 1). The specific fiber contents $\lambda_1 + \lambda_3 = |A\bar{y}|/|AD| \approx 0.15$, $\lambda_2 + \lambda_4 \approx 0.85$ are divided evenly between the fiber layers with stacking angles $\pm\varphi_\alpha$. The stacking scheme presented solves the design problem when utilizing the greatest possible fiber stacking angles (a question that plays an important part when fabricating the composite by winding).

Let us solve the problem of creating the same composite when utilizing a minimal fiber volume. The ray is $L = \{(0.5; 0.2)0.8/s : s \in [0.8; 0)\}$, see Fig. 1. The point B in Fig. 1 corresponds to the minimal value of s for which the problem (2.9), (2.6), and (2.8) is solvable. We now find $\pm\varphi_1 = \pm 90^\circ$, $\pm\varphi_2 = 0$, $\lambda_1 + \lambda_3 \approx 0.25$, $\lambda_2 + \lambda_4 \approx 0.75$. The fiber volume content is $s = 0.55$.

DESIGN OF FIBROUS COMPOSITES WITH GIVEN STRAIN-STRENGTH CHARACTERISTICS

1. Is the problem (2.9), (2.6), and (2.8) with the condition

$$M \equiv \max_{\alpha \in \Sigma} \{f_1(\varphi_\alpha, e_{ij}),\ f_2(\varphi_\alpha, \varphi_{\alpha+1}, e_{ij}),\ f_b(\varphi_\alpha, e_{ij})\} \leqslant \sigma \tag{2.10}$$

solvable ($\sigma = 1$).

2. If solvable, then indicate the set of its solutions.

The methods of solving the formulated problem are based on the results presented above. Let us consider the case when the fiber stacking angles take on a finite number of values $\{\varphi^\gamma\}_{\gamma=1}^m$. Let us introduce the set ($\sigma = 1$)

$$\mathcal{U}_2(\sigma) = \{(\varphi, \psi) \in \{\varphi^\gamma\}_{\gamma=1}^m : f_1(\varphi, e_{ij}) \leqslant \sigma,\ f_1(\psi, e_{ij}) \leqslant \sigma,\ f_2(\varphi, \psi, e_{ij}) \leqslant \sigma\}; \tag{2.11}$$

$$\mathcal{U}(\sigma) = \{\varphi \in \{\varphi^\gamma\}_{\gamma=1}^m : f_b(\varphi, e_{ij}) \leqslant \sigma\}. \tag{2.12}$$

The period of the composite configuration contains not more than m bonding fiber layers of different orientation. Each layer is characterized by a stacking angle φ_α and the specific fiber content λ_α while the whole structure is characterized by the vectors $\bar{\varphi} = (\varphi_1, \ldots, \varphi_m)$, $\bar{\lambda} = (\lambda_1, \ldots, \lambda_m)$. The vectors $\bar{\varphi}$, $\bar{\lambda} \in R^m$ ($\lambda_\alpha = 0$ corresponds to absence of a layer). Let us note that the indices α in φ_α and γ in φ^γ are possible values of the stacking angles and are different. The angles φ_α can take on any values from the set $\{\varphi^\gamma\}_{\gamma=1}^m$ while λ_α equal zero (when the given stacking is not utilized).

Let us examine two physically adjacent bonding layers (we give them the subscripts α, β). The vector $\bar{\lambda}$ in the parts corresponding to these layers is $\bar{\lambda} = (\ldots, \lambda_\alpha, 0, \ldots, 0, \lambda_\beta, \ldots)$. The strength conditions (2.11) and (2.12) are satisfied if and only if the adjacent stacking angles are

$$(\varphi_\alpha, \psi_\beta) \in \mathcal{U}_2(\sigma) \cap \mathcal{U}(\sigma). \tag{2.13}$$

Therefore, it is necessary to obtain an algorithm for constructing the bonding schemes $\bar{\lambda}$, $\bar{\varphi}$ for which condition (2.13) is satisfied for adjacent (i.e., separated by zeroes in writing $\bar{\lambda}$) coordinates of the vector $\bar{\varphi}$.

To obtain the algorithm it is sufficient to indicate how by having the vector fragment $(\lambda_1, \ldots, \lambda_\varkappa)$, $(\varphi_1, \ldots, \varphi_\varkappa)$, $\varkappa < m$ satisfying (2.13), to supplement it by the stacking angle $\psi \in \{\varphi^\gamma\}_{\gamma=1}^m$ corresponding to the condition:

$$\text{if } \lambda_{\varkappa+1} \neq 0, \text{ then } (\varphi_\varkappa, \psi) \in \mathcal{U}_2(\sigma) \cap \mathcal{U}(\sigma),$$
$$\text{if } \lambda_{\varkappa+1} = 0, \text{ then } \psi \text{ is arbitrary.}$$

One of the possible algorithms for the solution of the last problem is ($\varkappa$ enumerates the steps of the algorithm): 1. We set $\varkappa = 1$ and select an arbitrary integer $R \leq m$. 1.1. We select an arbitrary point $(x, y) \in \mathcal{U}_2(\sigma) \cap \mathcal{U}(\sigma)$. 1.2. We set $\varphi_1 = x$.

2. Among the points of the form $(\varphi_\varkappa, y)$ we seek those belonging to $\mathcal{U}_2(\sigma) \cap \mathcal{U}(\sigma)$. If there aren't any, stop. If there are, we set $\varphi_{\varkappa+1} = y$ and $\varkappa = \varkappa + 1$. If $\varkappa = R$, stop (we set $\varphi_{R+1} = \cdots = \varphi_m = 0$) if $\varkappa < R$, repeat step 2. The number R has the meaning of the number of nonzero elements in the vector $\bar{\lambda}$. By variating the selection we obtain different vectors $\bar{\varphi}$, $\bar{\lambda}$.

The arrangement of the coordinates in the vector $\bar{\varphi}$ obtained is arbitrary. Let us order its coordinates in the same order as in $\{\varphi^\gamma\}_{\gamma=1}^m$. For instance, let $\{\varphi^\gamma\}_{\gamma=1}^m$ be arranged in increasing order. Let N denote the commutation operator that arranges the coordinates of $\bar{\varphi}$ in increasing order: $(N\bar{\varphi})_1 \leqslant \ldots \leqslant (N\bar{\varphi})_m$. We act on the vector $\bar{\lambda}$ with the same operator. We satisfy condition (2.10) by application of the algorithm. It is still required to satisfy (2.9), (2.8), and (2.5). By virtue of Proposition 2 this condition is equivalent to the following: $N\bar{\lambda} \in \Lambda(\bar{y})$ where $\Lambda(\bar{y})$ is given by (1.6). Let us note that the conditions on $N\bar{\lambda}$ occurring here reduce just to the fact that zeroes are at definite $N\bar{\lambda}$ sites.

Let $\bar{v} \in R^m$, we define the vector $\operatorname{sgn}\bar{v}$ (the signature of $\bar{v}$) by it according to the rule: $(\operatorname{sgn}\bar{v})_i = 0$ if $v_i = 0$, $(\operatorname{sgn}\bar{v})_i = 1$ if $v_i \neq 0$. Let us write $\operatorname{sgn}\bar{v} \in \operatorname{sgn}\bar{w}$ if $(\operatorname{sgn}\bar{w})_i = 1$ $\forall i = 1, \ldots, m$ follows from $(\operatorname{sgn}\bar{v})_i = 1$.

Therefore, we must confirm the embedding $N\bar{\lambda} \Supset \operatorname{sgn}\bar{\lambda}$ for a certain $\bar{\lambda} \in \Lambda(\bar{y})$. Since the sum of the numbers $\lambda_\alpha \geq 0$ equals 1, then this condition is equivalent to $\bar{\lambda}\operatorname{sgn} N\bar{\lambda} = 1$ for a certain $\bar{\lambda} \in \Lambda(\bar{y})$. Substituting $\bar{\lambda}$ here in terms of the simplicial solution (see Proposition 2), we obtain

$$\sum_{\eta=1}^{M} \mu_\eta (\bar{s}_\eta(\bar{y}) \operatorname{sgn} N\bar{\lambda} - 1) = 0$$

for a certain set $\{\mu_\eta\}$. Since

$$\bar{s}_\eta(\bar{y}) \operatorname{sgn} N\bar{\lambda} \leqslant \sum_{\alpha=1}^{m} s_{\eta\alpha}(\bar{y}) = 1 \text{ and } \mu_\eta \geqslant 0, \text{ then}$$

$$\mu_\eta(\bar{s}_\eta(\bar{y})\,\mathrm{sgn}\,N\bar{\lambda} - 1) = 0 \quad \text{for all} \quad \eta = 1, \ldots, M.$$

Therefore, the question is reduced to a question about the existence of simplicial solutions for which

$$\bar{s}_\eta(\bar{y})\,\mathrm{sgn}\,N\bar{\lambda} = 1 \tag{2.14}$$

(since the μ_η cannot vanish simultaneously because $\sum_{\eta=1}^{M} \mu_\eta = 1$). Confirmation of the latter condition is easily realized in practice because of the finiteness of the number of simplicial solutions.

In other words, the procedure for solving the design problem consists of initiating different bonding schemes satisfying (2.11) and (2.12) by the algorithm until we find the bonding scheme satisfying (2.14). Afterwards, the action can understandably be continued in order to find other schemes also.

DESIGN OF A MAXIMAL STRENGTH COMPOSITE

We assumed $\sigma = 1$ above [see (2.11) and (2.12)]. According to the definition, σ is the value of the maximum in (2.10): $M = \sigma$. Correspondingly, $M = \sigma$ for $\sigma \leq 1$ has the physical meaning of the safety factor (indicating how much the values of the strength criterion M "are removed" from the limit value equal to one).

Solution of the problem $M \to \min$ upon satisfaction of the conditions (2.9), (2.8), (2.5) is performed on the basis of the algorithm described above. Only σ in the definitions (2.11) and (2.12) is not taken equal to 1 but in the form of a parameter increasing from 0 to 1, and the minimal value of σ is sought for which the problem (2.9), (2.8), (2.5), and (2.10) is solvable. Since the change in the sets (2.11) and (2.12) occurs discretely as σ grows from 0 to 1 for a finite number of stacking angle values, then the problem of finding the σ mentioned becomes practically fully solvable (an analogous case is examined in [16]).

STRUCTURE DESIGN

Composite technologies combine, in a natural manner, the possibilities of material and structure design (structures are understood to be bodies with distributed mechanical characteristics). Control of a composite microstructure permits obtaining, in principle, a body with a given average characteristic distribution, elastic, say, $\hat{a}_{ijk\ell}(\bar{x})$, $\bar{x} \in Q$ (Q is the domain occupied by the structure).

How to solve the problem in practice? This can be done on the basis of the methods elucidated. Let us take the relationships (1.1) or (2.1), let us compile the system S and solve it. The solution will depend on $\bar{x} \in Q$ as on a parameter. Afterwards, we solve the problem (1.4) [or (2.9)] with the right side $\bar{y} \in Y(\bar{x})$. We obtain the desired answer, a local composite configuration at a given point of the structure. The possibility of discretization of the parameter $\bar{x} \in Q$, as is necessary for practical solution of the problem, follows from Proposition 3 [under the condition $Y(\bar{x}) \in C(Q)$].

Example 2. Let it be required to fabricate a material with the following stiffness distribution: $\hat{a}_{1111} = (0.25-0.125x_1)\cdot 10^{11}$ Pa, $\hat{a}_{2222} = 0.25\cdot 10^{11}$ Pa, where $x_1 \in [0, 1]$. Let a fiber with the Young's modulus $E = 1.25\cdot 10^{11}$ Pa (fiberglass) be used. Symmetric bonding schemes of the type $\{\pm\varphi_\alpha\}$ are applied.

The solution of the system S for $s_0 = 1$ (physically not realizable) is $\bar{y}_0(x_1) = (0.2 - 0.1x_1;\ 0.2)$ to which the ray $L(x_1) = \{(1/s)(0.2 - 0.1x_1;\ 0.2):\ s \in [1, 0)\}$ corresponds. Let us set $\Phi_\alpha = [45°, 90°]$. In the case under consideration $\Gamma = \{(\eta, (1 - \sqrt{\eta})^2):\ \eta \in [0, \cos^4 45°]\}$ [Fig. 2, rays $L(x_1)$ are displayed there]. As is seen, the problem is solvable since $L(x_1) \cap \mathrm{conv}\,\Gamma \neq \varnothing$ for all $x_1 \in [0, 1]$, the solutions are not identical. The continuity condition for $\hat{a}_{ijk\ell}(\bar{x})$ [and $Y(\bar{x})$] was imposed above but in our case it retains the multiplicity of the solutions which is understandably a disadvantage and affords freedom in the selection of the technology for realization of the design. Without wishing to make the demonstration solution awkward, we extract the unique solution by requiring that the fiber volume content be minimal. Geometrically this means that points of the segment [c, d] are con-

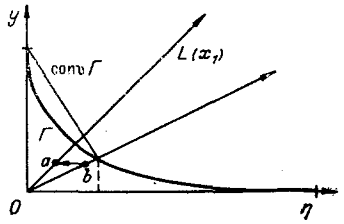

Fig. 2

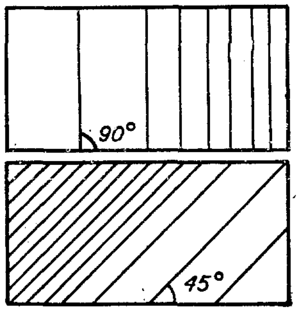

Fig. 3

sidered. We find that the fiber volume content in the design is

$$s(x_1) = 0.8 - 0.3x_1.$$

Afterwards we solve the problem (2.9), (2.6), (2.8) with the right side $(1/s(x_1))\bar{y}_0(x_1)$: the stacking angles are $\pm\varphi_1(x_1) \equiv 90°$, $\pm\varphi_2(x_1) \equiv \pm 45°$ the specific fiber content in the bonding layers is $\lambda_1(x_1) + \lambda_3(x_1) = 0.205x_1$ and $\lambda_2(x_1) + \lambda_4(x_1) = 1 - 0.205\, x_1$ (the specific contents are divided equally between layers with angles $\pm\varphi_\alpha$).

The fiber volume contents in the bonding families are $\lambda_1(x_1)s(x_1) = 0.205x_1(0.8 - 0.3x_1)$, $\lambda_2(x_1)s(x_1) = (1 - 0.205x_1)(0.8 - 0.3x_1)$.

The required distribution of the fiber stacking density achieved by controlling the stacking intensity is represented in Fig. 3 (families with stacking angles $\varphi_1 = 90$ and $\varphi_3 = 45°$ are represented).

ELECTRONIC COMPUTER APPLICATION TO SOLVE DESIGN PROBLEMS

The low dimensionality of the appropriate problem about convex combinations is used substantially to solve the design problem in the examples presented of the graphical solution. In cases occurring in practice the dimensionality [number of equations in (1.4)] is on the order of 10. In the case of fibrous composites the dimensionality is 4 and higher (because of the introduction of additional variables; see later). The problem of convex combinations is a typical convex analysis problem for which the ineffectiveness of direct methods of solution is characteristic in connection with the very rapid growth of the volume of calculations as the dimensionality of the problem increases. To obtain particular solutions of (1.4) or (2.9), (2.6), (2.8) the simplex method turns out to be effective. An algorithm is proposed in [22] for the construction of the general solution $\Lambda(\bar{y})$ based on sequential satisfaction of the equations in the problem on convex combinations (based on constructing a set of simplicial solutions of the one-dimensional problem at each step, as is realized explicitly, and by making the calculation process purposeful, affords a possibility of performing computations for the number of equations that occurs in practice). The algorithm is realized in the form of programs for electronic computers that display their operability on model problems [23, 24].

UTILIZATION OF DIFFERENT SPECIES OF FIBERS (HYBRID COMPOSITES)

The functionals $R_1(\overline{\varphi}), \ldots, R_4(\overline{\varphi}), J_1(\overline{\varphi}), J_2(\overline{\varphi})$ acquire the form

$$R_1(\overline{\varphi}) = \sum_{\delta=1}^{s*} \sum_{\alpha=1}^{m} E_\delta \cos^4 \varphi_\alpha \lambda_{\alpha\delta} \quad \text{etc.},$$

$$J_1(\overline{\varphi}) = \sum_{\delta=1}^{s*} \sum_{\alpha=1}^{m} \beta_\delta \cos^2 \varphi_\alpha \lambda_{\alpha\delta} \quad \text{etc.},$$

$$\lambda_{\alpha\delta} \geqslant 0, \quad \sum_{\delta=1}^{s*} \sum_{\alpha=1}^{m} \lambda_{\alpha\delta} = 1,$$

where s^* is the number of species of fibers being utilized, and $\lambda_{\alpha\delta}$ is the specific content (referred to s) of δ-th species fiber in a fiber layer with stacking angle φ_α. The functionals $R_1(\bar{\varphi})$, ..., $R_4(\bar{\varphi})$ and $J_1(\bar{\varphi})$, $J_2(\bar{\varphi})$ here become independent. The problem dimensionality grows to six. The solvability condition is: the solution $\bar{y}$ of the system S belongs to the convex hull of the set

$$\Gamma = \{\bar{x} \in R^6\colon\ x_1 = E\cos^4\varphi, \ldots, x_5 = \beta\cos^2\varphi, \ldots, \\ \varphi \in \Phi_\alpha,\quad (E, \beta) \in \{(E_\delta, \beta_\delta)\}_{\delta=1}^{s^*}\}.$$

Growth of the dimensionality of the problem and the set Γ broadens the class of possible properties of the composites.

3. DESIGN OF PLATES. LAMINAR PLATES

The methods developed in [25] (see also the bibliography in [26]) permit explicit expressions to be obtained for the stiffnesses of laminar plates and formulation of the problem of designing laminar plates with given stiffnesses. In the case under consideration an equation occurs of the form

$$\int_0^1 \bar{f}(\bar{u}(y), y)\, dy = \bar{y}. \tag{3.1}$$

A description is given in [27] for the set of possible values of the average characteristics of laminar plates (in the set of materials with nonnegative mechanical characteristics), methods of numerical solution of (3.1) are discussed in [27, 28], and examples are examined. Let us note that the problem reduces to that considered in Sec. 2 for a large quantity of layers, see [29].

FIBROUS PLATES

A number of computations is carried out in [30] for the local stresses in a plate of fibrous configuration that permit giving an estimate of the stresses in the binder, and values of the average stiffnesses are presented. In conjunction with [31] this permits formulation of the design problem.

On the whole, a study of the methods of solving problems of designing composite plates with given characteristics has only started. The appearance of the argument y in the function $f(\bar{u}, y)$ in this case does not hinder application of the methods developed above but sharply raises the dimensionality of the problems for a numerical computation.

LAMINAR MEMBRANES, COATINGS, ETC.

The characteristics of laminar coatings and partitions (see [32, 33]) are given, as a rule, in the form of certain kinds of means, which permits utilization of the methods elucidated for their design.

LITERATURE CITED

1. A. Bensoussan, J. L. Lions, and G. Papanicolaou, Asymptotic Analysis for Periodic Structures, North-Holland, Amsterdam (1976).
2. N. S. Bakhvalov and G. P. Panasenko, Averaging of Processes in Periodic Media [in Russian], Nauka, Moscow (1984).
3. A. N. Tikhonov and V. Ya. Arsenin, Methods of Solving Incorrect Problems [in Russian], Nauka, Moscow (1986).
4. J. L. Lions, Optimal Control of Systems Governed by Partial Differential Equations, Springer-Verlag, New York (1971).
5. V. G. Litvinov, Optimization in Elliptic Boundary Value Problems with Applications to Mechanics [in Russian], Nauka, Moscow (1987).
6. G. P. Panasenko, "Strength of spatially-bonded composite materials," Vestn. Mosk. Univ., Ser. 15. Vychislit. Mat. Kibern., No. 1 (1985).
7. A. G. Kolpakov, "Averaging of certain systems of ordinary differential equations," Mat. Sb., 119, No. 4 (1982).
8. A. G. Kolpakov, "On determination of certain effective characteristics of composite materials," Annotated Reports. 5th All-Union Congress on Theor. and Appl. Mechanics [in Russian], Nauka, Alma-Ata (1981).

9. A. G. Kolpakov, "Effective thermoelastic characteristics of an inhomogeneous material," Din. Sploshnoi Sredy, No. 49, Inst. Gidrodin. Sib. Otd. Akad. Nauk SSSR (1981).
10. V. V. Alekhin, B. D. Annin, and A. G. Kolpakov, Synthesis of Laminar Materials and Structures [in Russian], Inst. Gidrodin., Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1988).
11. A. G. Kolpakov and S. I. Rakin, "On the problem of synthesis of a composite material of one-dimensional configuration with given characteristics," Prikl. Mekh. Tekh. Fiz., No. 6 (1986).
12. V. V. Alekhin and A. G. Kolpakov, "Synthesis of laminar elastic bodies and materials," in: Annotated Reports. 6th All-Union Congress on Theor. and Appl. Mechanics [in Russian], Tashkent (1986).
13. R. T. Rockafeller, Convex Analysis [Russian translation], Mir, Moscow (1973).
14. A. G. Kolpakov and S. I. Rakin, "Problem of synthesis of a composite of one-dimensional configuration in a given class of materials," Din. Sploshnoi Sredy, No. 78, Inst. Gidrodid., Sib. Otd. Akad. Nauk SSSR (1986).
15. A. G. Kolpakov, "Averaging in a second order ordinary differential equation," Differents. Uravn., 18, No. 10 (1982).
16. B. D. Annin and A. G. Kolpakov, "Design of laminar composites with given strain-strength characteristics," Mekh. Komposit. Mater., No. 1 (1987).
17. B. D. Annin and A. G. Kolpakov, "Design of laminar composites with given strain-strength characteristics," in: Abstracts of Reports. 6th All-Union Conf. on Polymer and Composite Materials [in Russian], Zinatne, Riga (1986).
18. K. A. Lur'e and A. V. Cherkaev, "Regularization of optimal design problems of inhomogeneous bodies by using composite materials," in: Annotated Reports. 5th All-Union Congress on Theor. and Appl. Mech., [in Russian], Nauka, Alma-Ata (1981).
19. Yu. N. Rabotnov, Mechanics of a Deformable Solid [in Russian], Nauka, Moscow (1979).
20. J. L. Lions, "Remarks on certain computational aspects of the homogenization method in composite materials," in: Calculation Methods in Mathematics, Geophysics, and Optimal Control [Russian translation], Nauka, Novosibirsk (1978).
21. A. G. Kolpakov, "Average characteristics of laminar composites (numerical algorithm)," in: Numerical Methods of Solving Elasticity and Plasticity Problems: Materials of the 9th All-Union Conf. [in Russian], Inst. Theor. Appl. Mech., Sib. Branch, USSR Acad. Sci., Novosibirsk (1986).
22. A. G. Kolpakov, "Average strength criterion of a binder of fibrous composites," Prikl. Mekh. Tekh. Fiz., No. 2 (1988).
23. A. G. Kolpakov, "Problem of design of fibrous composites with given characteristics," in: Materials, 6th All-Union Conf. on Composite Materials [in Russian], Vol. 1, Erevan (1987).
24. A. G. Kolpakov, "Strain-strength characteristics of laminar and fibrous composites, Analysis and design," in: Abstracts of Reports. 1st All-Union Symp., "Mechanics and Physics of Composite Material and Structure Rupture," [in Russian], Uzhgorod (1988).
25. D. Gailleria, "Thin elastic and periodic plates," Math. Meth. Eng. Sci., No. 12 (1986).
26. A. L. Kalamkarov, B. A. Kudryavtsev, and V. Z. Parton, "Asymptotic method of averaging in the mechanics of composites of regular configuration," in: Science and Engineering Surveys., Mechanics of a Deformable Solid [in Russian], Vol. 19, VINITI, Moscow (1987).
27. A. G. Kolpakov, "Analysis and design of laminar plates," Prikl. Mekh. Tekh. Fiz., No. 4 (1989).
28. A. G. Kolpakov, "Strength characteristics of laminar structures (analysis and design)," in: Abstracts of Reports, 3rd All-Union Sci.-Tech. Conf. "Dynamics and Strength of Automobiles" [in Russian], Moscow (1988).
29. A. G. Kolpakov, "Effective stiffnesses of composite plates," Prikl. Mat. Mekh., 46, No. 6 (1982).
30. V. Z. Parton, A. L. Kalamkarov, and A. G. Kolpakov, "On analysis of high-modulus cross-bonded composite shells," Mekh. Komposit. Mater., No. 1 (1989).
31. A. L. Kalamkarov, "On determination of the effective characteristics of reticulate shells and plates of periodic configuration," Izv. Akad. Nauk SSSR, Mekh. Tverd. Tela, No. 2 (1987).
32. A. G. Kolpakov and I. G. Kolpakova, "Heat conduction through the boundary with nonlocal characteristics," Differents. Uravn., 21, No. 9 (1985).
33. A. G. Kolpakov, "On the asymptotic of the first boundary value problem for elliptic equations in a domain with a thin coating," Sib. Mat. Zh., 29, No. 6 (1988).